Register No : ____________________

# THE KAVERY ENGINEERING COLLEGE

*(An Autonomous Institution)*

B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, NOV /DEC 2024

*Fourth Semester*

*Chemical Engineering*

MA3451–Transform Techniques

*Regulations - 2021*

*Duration : 3 Hrs* *Maximum : 100 Marks*

*PART – A (ANSWER ALL QUESTIONS) (Marks 10*2=20)*

| *Q.No* | *Questions* | *BLT* | *CO* | *Marks* |
|---|---|---|---|---|
| 1. | If $\vec{r} = x\vec{i} + y\vec{i} + z\vec{k}$, then find div $\vec{r}$. | UN | 1 | 2 |
| 2. | Find the value of '$a$'so that the vector $\vec{f} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+az)\vec{k}$ is solenoidal. | UN | 1 | 2 |
| 3. | State Dirichlet's conditions for Fourier series. | UN | 2 | 2 |
| 4. | Find $b_n$ in the expansion of $x$ as a Fourier series in $(-\pi,\pi)$ | UN | 2 | 2 |
| 5. | State the Fourier integral theorem. | RE | 3 | 2 |
| 6. | If $F[f(x)] = F(s)$, then prove that $F[f(ax)] = \frac{1}{a}F\left(\frac{s}{a}\right), a > 0.$ | RE | 3 | 2 |
| 7. | State sufficient conditions for existence of Laplace transform. | RE | 4 | 2 |
| 8. | If $L[f(t)] = F(s)$, then show that $L[e^{at} f(t)] = F(s-a)$. | UN | 4 | 2 |
| 9. | Prove that $Z[a^n] = \frac{z}{z-a}$, if $\|z\| > \|a\|$. | RE | 5 | 2 |
| 10. | State the initial value theorem in Z-transform. | RE | 5 | 2 |

*PART – B (ANSWER ALL QUESTIONS) (Marks 5*16 = 80)*

| *Q.No* | | | *Questions* | *BLT* | *CO* | *Marks* |
|---|---|---|---|---|---|---|
| 11. | a | i | Find the directional derivative of $xy^2 + yz^3$ at $(2,-1,1)$ in the direction of $\vec{i} + 2\vec{j} + 2\vec{k}$. | AP | 1 | 8 |
| | | ii | Find the value of the constant a, b, c, so that the vector $\vec{F} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational. | AP | 1 | 8 |
| | | | Or | | | |
| | b | | Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ over the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$. | AP | 1 | 16 |

12. a i Find Fourier series of f(x) = $x^2$ in $(-\pi, \pi)$ hence deduce AN 2 8

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots=\frac{\pi^2}{6}$$

ii Find the Fourier series up to the second harmonic for $y = f(x)$ in $(0, 2\pi)$ defined by the table values given below AN 2 8

| x | 0 | π/3 | 2π/3 | π | 4π/3 | 5π/3 | 2π |
|---|---|---|---|---|---|---|---|
| y | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1 |

Or

b i Find Fourier series of $f(x) = \begin{cases} l-x\,, 0<x<l \\ 0 \quad , l<x<2l \end{cases}$ AN 2 8

ii Find half range cosine series of f(x) = x in $(0, \pi)$ and hence deduce $\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+\cdots$ AN 2 8

13. a Find the Fourier transform of $(x) = \begin{cases} 1-x^2, |x|<1 \\ 0, otherwise \end{cases}$. AP 3 16

Hence prove that

( i) $\int_0^\infty \left[\frac{sins-scoss}{s^3}\right] cos\frac{s}{2}\, ds = \frac{3\pi}{16}$

(ii) $\int_0^\infty (\frac{scos\, s-\sin s}{s^3})^2\, ds = \frac{\pi}{15}$

Or

b i Find the Fourier sine transform of $e^{-ax}$ and hence evaluate $\int_0^\infty \frac{x^2dx}{(x^2+a^2)\ (x^2+b^2)}$. AP 3 8

ii Find Fourier cosine and sine transform of $e^{-x}$ and hence deduce $F_s\left[\frac{x}{1+x^2}\right]$ and $F_c\left[\frac{1}{1+x^2}\right]$. AP 3 8

14. a Find the Laplace transform of the function $f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$ with $f(t+2a)=f(t)$ AP 4 16

Or

b i Solve $(D^2+4D+4)y=e^{-t}$ given that $y(0)=y'(0)=0$ using Laplace transformation. AP 4 8

ii Using convolution theorem, find $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$. AP 4 8

| | | | | | | |
|---|---|---|---|---|---|---|
| 15. | a | i | Find Z transform of Cos $n\theta$ and Sin $n\theta$. | EV | 5 | 8 |
| | | ii | Using Z- transform, solve $y(n+2)-3\,y(n+1) + 2\,y(n) = 2^n$, given that $y(0) = 0$, $y(1) = 0$. | EV | 5 | 8 |
| | | | Or | | | |
| | b | i | Find $Z^{-1}\left[\frac{z^3}{(z-1)^2\,(z-2)}\right]$ using partial fraction method. | EV | 5 | 8 |
| | | ii | Using convolution theorem, evaluate $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$. | EV | 5 | 8 |